1. Prove for any sets A, B, C,

(i) $A - B = A \cap B'$

(ii) $A \subseteq B \Rightarrow B' \subseteq A'$

(iii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iv) $A - (B - C) = (A - B) \cup (A - C)$

2. In a survey of 60 people it was found that 25 people read newspaper A, 26 read newspaper B, 26 read newspaper C, 9 read both A & C, 11 read both A & B, 8 read both B & C and 3 read all the three newspapers A, B & C. Find the no. of people who read

(i) at least one of the three newspapers

(ii) exactly one newspaper

3. Find the domain & range of the relations

(i) $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

(ii) $R = \{(x, x^3) : x$ is a prime no. less than $10\}$

4. Let R be the relation on the set $\mathbb{Q}$ of ~~integers defined by~~ rational nos. defined by $R = \{(a, b) : a, b \in \mathbb{Q}, a - b$ is an integer$\}$. Show that

(i) $(a, a) \in R \quad \forall \ a \in \mathbb{Q}$

(ii) $(a, b) \in R \Rightarrow (b, a) \in R \quad \forall \ a, b \in \mathbb{Q}$

(iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R \quad \forall \ a, b, c \in \mathbb{Q}$

5. Let R be the relation on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by 7. Prove that

(i) $(x, x) \in R \quad \forall \ x \in Z$

(ii) $(x, y) \in R \Rightarrow (y, x) \in R \quad \forall \ x, y \in Z$

(iii) $(x, y) \in R$ & $(y, z) \in R \Rightarrow (x, z) \in R \quad \forall \ x, y, z \in Z$

6. Find the domain and range of the functio

(i) $f(x) = \sqrt{9 - x^2}$

(ii) $f(x) = \frac{1}{1 - x^2}$

(iii) $f(x) = \frac{x - 2}{3 - x}$

(iv) $f(x) = \frac{x^2}{1 + x^2}$

7. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

8. Prove that $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$

9. Prove that
   (i) $\tan 82\frac{1}{2}^\circ = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$
   (ii) $\cot 7\frac{1}{2}^\circ = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

10. Prove that (in $\triangle ABC$)
    $(b^2-c^2)\cot A + (c^2-a^2)\cot B + (a^2-b^2)\cot C = 0$

11. Prove that (in $\triangle ABC$)
    $a^3\cos(B-C) + b^3\cos(C-A) + c^3\cos(A-B) = 3abc$

12. In any $\triangle ABC$, Prove that
    $a\cos A + b\cos B + c\cos C = \frac{8\Delta^2}{abc}$

13. In any $\triangle ABC$, Prove that
    $\left(\frac{b^2-c^2}{a^2}\right)\sin 2A + \left(\frac{c^2-a^2}{b^2}\right)\sin 2B + \left(\frac{a^2-b^2}{c^2}\right)\sin 2C = 0$

~~14. A lamp-post is situated at the middle point M of side AC of a $\triangle ABC$ triangular~~

14. Solve: $\cos x + \cos 3x - \cos 2x = 0$

15. Solve: $\cos x \cos 2x \cos 3x = \frac{1}{4}$

16. Solve: $\sqrt{3}\cos x + \sin x = 1$

17. Solve: $\sin^2 x - \cos x = \frac{1}{4}$

18. Solve: $(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 2$

19. Find the max. and min. values of
    $7\cos x + 24\sin x + 5$

20. If the angle θ is divided into two parts such that tangent of one part is λ times the tangent of other and ϕ is their difference then show that
    $\sin\theta = \frac{\lambda+1}{\lambda-1}\sin\phi$

21. If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$, prove that $xy + yz + zx = 0$

22. $m\sin\theta = n\sin(\theta+2\alpha)$, prove that
    $\tan(\theta+\alpha)\cot\alpha = \frac{m+n}{m-n}$

23. Show that $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15} = \frac{1}{16}$

24. If $\tan\theta_2 = \sqrt{\frac{a-b}{a+b}}\tan\frac{\phi}{2}$, prove that

$$\cos\theta = \frac{a\cos\phi + b}{a + b\cos\phi}$$

25. If α and β are the solutions of $a\tan x + b\sec x = c$, then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}$

26. Using principle of mathematical induction, prove that

$$\frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4} \quad \forall n \in \mathbb{N}$$

27. Using principle of mathematical induction prove that

$$\sin x + \sin 3x + \cdots + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x} \quad \forall n \in \mathbb{N}$$

28. Using mathematical induction, p.t.

$$\sin\theta + \sin 2\theta + \sin 3\theta + \cdots + \sin n\theta = \frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}\sin\left(\frac{n+1}{2}\right)\theta$$

29. Using mathematical induction prove that $x^{2n-1} + y^{2n-1}$ is divisible by $x+y$ $\forall n \in \mathbb{N}$

30. Let $P(n)$ be the statement: $2^n \geq 3n$.
If $P(r)$ is true, show that $P(r+1)$ is true.
Do you conclude that $P(n)$ is true $\forall n \in \mathbb{N}$?

31. If $(x+iy)^{1/3} = a+ib$, show that
$\frac{x}{a} + \frac{y}{b} = 4(a^2-b^2)$

32. Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real

33. If α and β are different complex numbers with $|\beta| = 1$, show that
$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

34 If $|z_1| = |z_2| = \cdots = |z_n| = 1$, prove that

$$|z_1 + z_2 + \cdots + z_n| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \cdots + \frac{1}{z_n} \right|$$

35 Find the square root of $-5+12i$

36. Write $\frac{1+7i}{(2-i)^2}$ in polar form

37. Write the complex no. $\frac{1-i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in polar form.

38. Solve: $2x^2 - (3+7i)x - (3-9i) = 0$

39. Solve: $x^2 - (7-i)x + (18-i) = 0$

40. Solve: $\frac{|x+3| + x}{x+2} > 1$

41. Solve $\frac{1}{|x|-3} \leq \frac{1}{2}$

42 Solve: $|x+1| + |x-2| > 3$

43 Solve graphically: $2x+3y \leq 6$, $x \leq 5$, $y \geq 7$, $x, y \geq 0$

44. If the first and the nth terms of a GP are a & b respectively and if P is the product of the first n terms, prove that $P^2 = (ab)^n$

45 The sum of three nos. in GP is 56. If 1, 7 and 21 are subtracted from the numbers respectively, resulting numbers form an AP. find the numbers.

46 If S be the sum, P, the product and R the sum of reciprocals of n terms of a GP, prove that $\left(\frac{S}{R}\right)^n = P^2$

47. Sum to n terms:

$4 + 44 + 444 + \cdots\cdots$

48. If a & b are roots of $x^2 - 3x + p = 0$ and c & d are roots of $x^2 - 12x + q = 0$ where a, b, c, d are in GP then prove that $(q+p) : (q-p) = 17 : 15$

49. Sum the series upto n terms:

(i) $1^2 + (1^2+2^2) + (1^2+2^2+3^2) + \cdots$

(ii) $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots\cdots$

(iii) $5 + 7 + 13 + 31 + 85 + \cdots$

(iv) $\frac{1}{1\cdot 6} + \frac{1}{6\cdot 11} + \frac{1}{11\cdot 16} + \frac{1}{16\cdot 21} + \cdots + \frac{1}{(5n-4)(5n+1)}$

50. If S_1, S_2, S_3 are the sums of first n natural nos, their squares and their cubes respectively, show that

$$9S_2^2 = S_3(1+8S_1)$$

Level-2

1. Prove that $5\cos x + 3\cos\left(x+\frac{\pi}{3}\right) + 3$ lies between -4 and 10

2. Prove that

$$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n = \begin{cases} 2\cot^n\left(\frac{A-B}{2}\right) & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

3. Find the sum of all solutions of $\cos x \cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}$, $x \in [0, 6\pi]$

4. Solve: $8^{1 + |\cos x| + \cos^2 x + |\cos^3 x| + \cdots \infty} = 4^3$

5. Prove by mathematical induction that $(1+x)^n \geq 1+nx \quad \forall n \in N$ where x is +ve.

6. Solve: $z^2 = \bar{z}$

7. Find the no. of solutions of $z^2 + |z|^2 = 0$

8. Find the value of x and y for which complex nos. $-3+ix^2y$ and x^2+y+4i are conjugate of each other

9. Solve $\frac{|2x-3|}{|x+2|} \leq \frac{1}{2}$

10. Solve: $|x| + |x-1| + |x-2| \geq 6$

11. Let R be a relation on $N \times N$ defined by $(a,b) R (c,d) \Leftrightarrow a+d = b+c \quad \forall (a,b), (c,d) \in N \times N$. Show that

i. $(a,b) R (a,b) \quad \forall (a,b) \in N \times N$

ii. $(a,b) R (c,d) \Rightarrow (c,d) R (a,b) \quad \forall (a,b), (c,d) \in N \times N$

iii. $(a,b) R (c,d)$ & $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$ $\forall (a,b), (c,d), (e,f) \in N \times N$

12. Find the domain of $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$

13. Find the domain of $f(x) = \log_4 \{ \log_5 (\log_3 (18x - x^2 - 77)) \}$

14. Find the domain of $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

15. Find the domain and range of $f(x) = \frac{1}{2 - \sin 3x}$

16. Find the domain and range of $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

17. Prove that

$$2^{\frac{1}{4}} \cdot 4^{\frac{1}{8}} \cdot 8^{\frac{1}{16}} \cdot 16^{\frac{1}{32}} \cdots \infty = 2$$

18. If $x = 1 + a + a^2 + \cdots \infty$ where $|a| < 1$ & $y = 1 + b + b^2 + \cdots \infty$ where $|b| < 1$, prove that

$$1 + ab + a^2b^2 + \cdots \infty = \frac{xy}{x + y - 1}$$

19 Find the sum of the series

$$1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + (n-1) \cdot 2 + n \cdot 1$$

20 Find the sum to n terms the series

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \cdots$$